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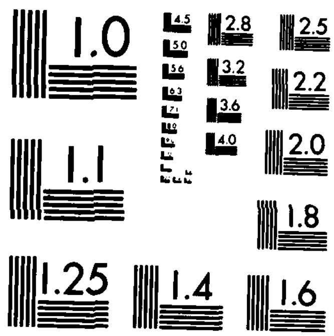

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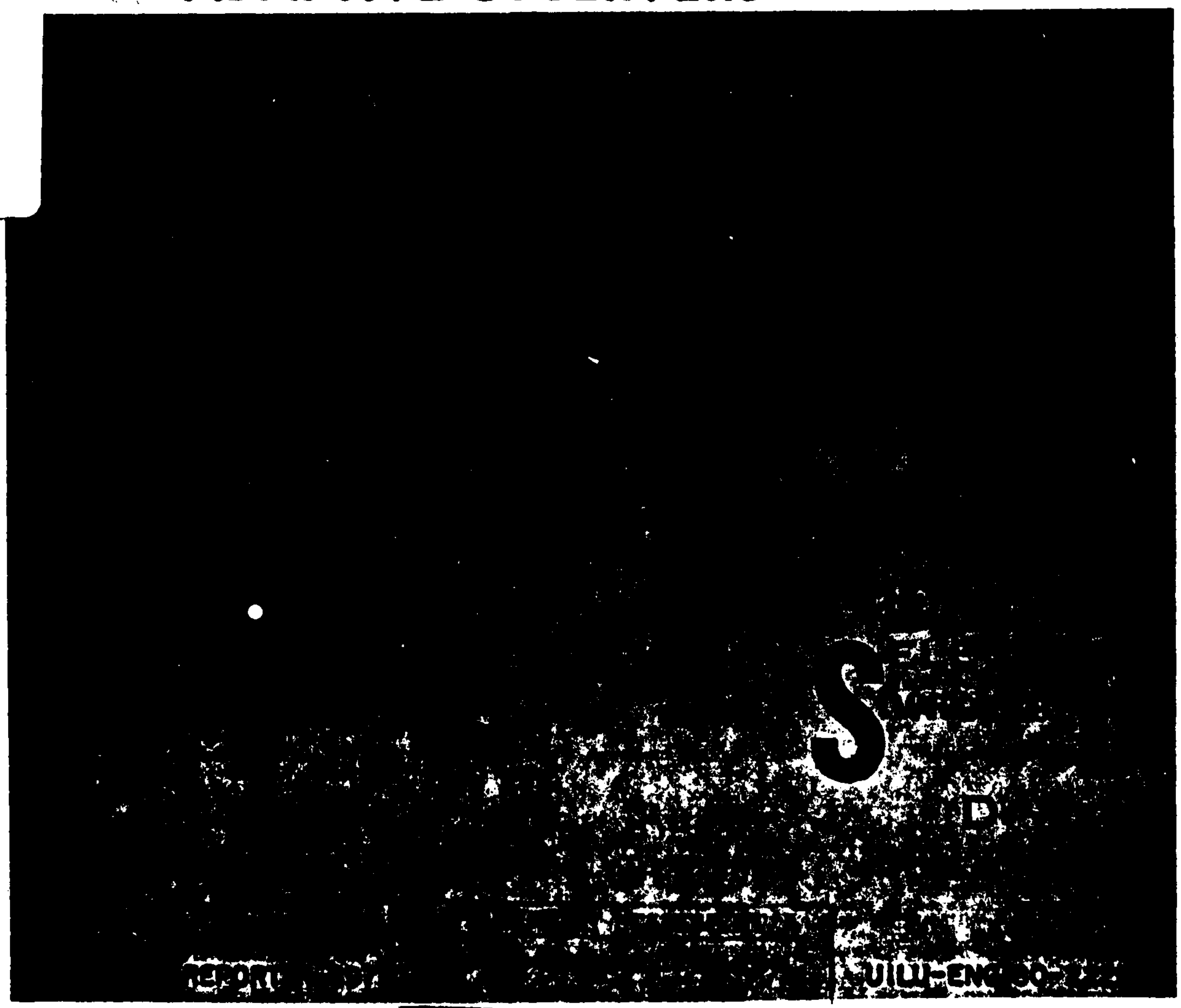

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by

P. A. Ioannou and P. V. Kokotovic

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ERROR BOUNDS FOR MODEL-PLANT MISMATCH IN IDENTIFIERS AND ADAPTIVE OBSERVERS*

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ABSTRACT

The stability properties of six different adaptive schemes with respect to model order error are analyzed. Bounds on parameter identification and state errors are established. All adaptive schemes considered are robust in the sense that the error is of order of the "speed ratio μ" between the modeled slow phenomena vs. the neglected fact. The dependence of the error on the input signal is shown to be crucial. The bounds obtained indicate possibilities for reducing the error by a proper choice of the input signal.

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1. INTRODUCTION

Global stability properties of model reference adaptive systems [1]-[10] are guaranteed under the "matching assumption" that the model order is not lower than the order of the unknown plant. Since this restrictive assumption is likely to be violated in applications, it is important to determine the robustness of adaptive schemes with respect to such modeling errors. Recently several attempts have been made to formulate and analyze reduced order adaptive identifiers [11]. The results of such studies depend on the characterization of the model-plant mismatch.

In this paper we examine stability properties and performance of various types of identifiers and adaptive observers [1]-[10] when the model-plant mismatch is due to a fast ("parasitic") part of the plant, and the order of the model is equal to the order of the slow ("dominant") part of the unknown plant. We express our results in terms of a "speed ratio" μ of the slow versus the fast phenomena. Scalar μ is small and positive and $\mu \downarrow 0$ means that the fast part of the plant reaches its steady-state instantaneously, that is the plant order reduces to that of its slow part. The fictitious "reduced-order" plant is thus obtained when in the actual plant $\mu > 0$ is replaced by $\mu = 0$.

This singular perturbation approach is a convenient parameterization of the model-plant mismatch. In our formulation adaptive observers are designed for the reduced order plants, but they are applied to the actual plants. In Section 2 we derive a singularly perturbed state space realization of the plant and give a statement of the problem. In Section 3 we analyze the stability properties of an identification scheme [1], [2] employing a lower order model

and we obtain bounds for parameter identification and state errors. In Section 4 we analyze the stability properties of reduced order minimal adaptive observers [5]-[7] applied to actual plants and obtain bounds for parameter and observation errors. The stability properties of a nonminimal adaptive observer [4], [8] designed for a lower order plant and applied to the actual higher order plant are analyzed (Section 5) and bounds are obtained on the parameter identification and output errors. In Section 6 similar results are obtained for the parametrized adaptive observer [9]. A qualitative analysis based on these bounds is given in Section 7 illustrated with computer simulation results. Particularly important is the sensitivity of the parameter identification error with respect to the excitation input signal.

2. PROBLEM STATEMENT

Systems possessing slow and fast parts can be represented in the explicit singular perturbation form

$$\dot{x} = A_{11}x + A_{12}x_f + B_1u \tag{2.1}$$

$$\mu\dot{x}_f = A_{21}x + A_{22}x_f + B_2u \tag{2.2}$$

$$y = c'x \tag{2.3}$$

where x, x_f are n and m vectors respectively, u is an r control vector and μ is a small positive parameter associated with the presence of "parasitic" elements, such as time constants, masses, etc. [14]. The matrices $A_{11}, A_{12}, A_{21}, A_{22}, B_1$ and B_2 have appropriate dimensions.

Without altering the input-output characteristics of the system we will use the transformation [13] $\eta = x_f + Lx + A_f^{-1}B_f u$ and analyze the equivalent representation

$$\dot{x} = Ax + Bu + H\eta \tag{2.4}$$

$$\mu\dot{\eta} = A_f\eta + \mu A_f^{-1}B_f\dot{u} \tag{2.5}$$

$$y = c'x \tag{2.6}$$

where

$$A = A_{11} - A_{12}L, \quad A_f = A_{22} + \mu LA_{12}, \quad B_f = B_2 + \mu LB_1$$

$$B = B_1 - A_{12}A_f^{-1}B_f, \quad H = A_{12}$$

L satisfies the algebraic equation

$$A_{22}L - A_{21} + \mu LA_{12}L - \mu LA_{11} = 0 \tag{2.7}$$

Approximate expressions for L, A, A_f and B are

$$L = A_{22}^{-1}A_{21} + 0(\mu) \tag{2.8}$$

$$A = A_{11} - A_{12}A_{22}^{-1} + 0(\mu), \; A_f = A_{22} + 0(\mu), \tag{2.9}$$

$$B_j = B_2 + 0(\mu), \; B = B_1 - A_{12}A_{22}^{-1}B_2 + 0(\mu) \tag{2.10}$$

Representation (2.4) - (2.6) containing is found to be convenient for getting tighter error bounds and clarifying the dependence of the error on the characteristics of the input.

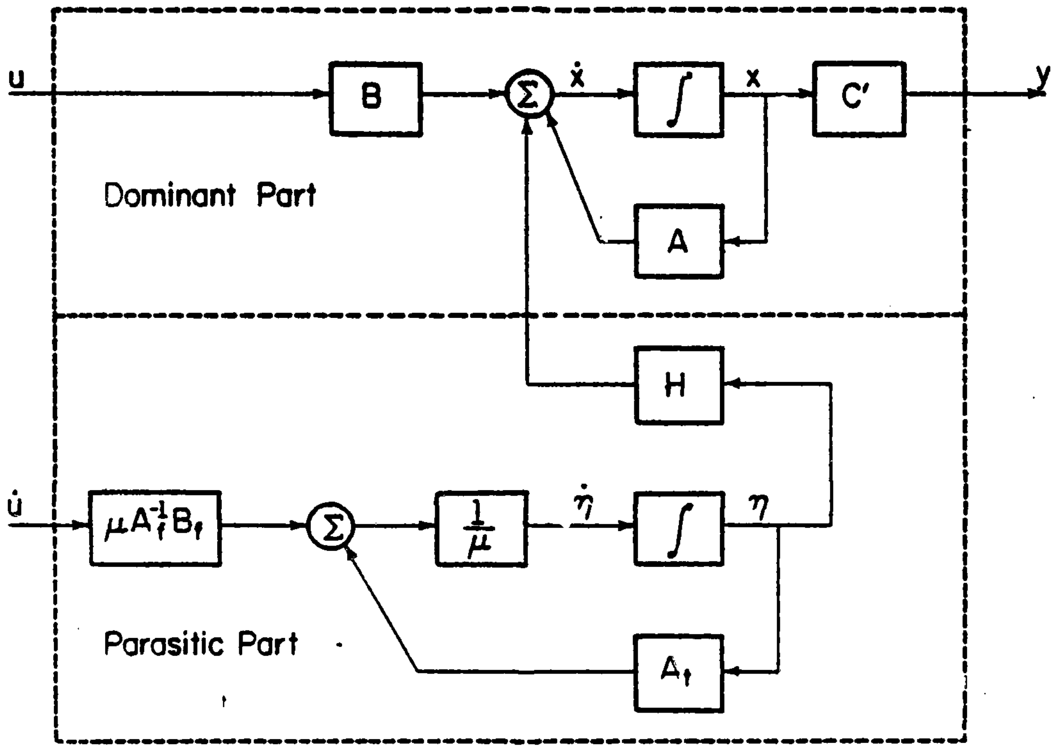

Fig. 1. Representation of the plant based on (2.4) - (2.6).

The part of the system described by (2.4) and (2.6) will be referred to as the dominant part of the plant whereas (2.5) will be called the parasitic part of the plant. Suppose that an adaptive scheme is designed for the n-order dominant part assuming that there are no parasitics ie $H\eta = 0$. This scheme is then applied to the actual plant with parasitics. The purpose of this paper is to examine the robustness of the scheme with respect to the parasitic part of the plant and to obtain bounds on the parameter and output or state errors.

Throughout the paper the following assumptions are made:

(i) A is stable and A_f is asymptotically stable

(ii) The order of the dominant part of the plant is known

(iii) The tripple (A,B,C) is completely controllable and completely observable

(v) The only available signals are $u(t)$ and $y(t)$

(vi) $u(t)$ and $\dot{u}(t)$ are piecewise continuous bounded functions of time.

It will be shown in the following sections that the stability of several adaptive algorithms in the presence of parasitics is equivalent to the stability of a linear time-varying equation with a parasitic input

$$\dot{Z}(t) = A_\eta(t)Z(t) + \bar{H}\eta(t) \tag{2.11}$$

where $Z(t)$ is a composite error vector. It should be pointed out that (2.11) is not input η to state $Z(t)$ linear because $A_\eta(t)$ depends on x which in turn depends on η. This dependence will be explored for each particular scheme. Our approach is to first derive conditions under which the homogeneous part of (2.11) is uniformly asymptotically stable (u.a.s.) for each η of interest.

After these conditions are found, Lemma 1 is used to obtain bounds on Z(t).

<u>Lemma 1</u>: If the homogeneous part of (2.11) is u.a.s. then Z(t) is bounded. A bound on the norm of Z(t) as $t \to \infty$ is of order of μ and is given by

$$\lim_{t\to\infty} \|Z(t)\| \leq \mu\gamma \frac{m_1}{m_2}\frac{\alpha_1}{\alpha_2}\|\bar{H}\| \, \|A_f^{-1}B_f\| \tag{2.12}$$

<u>Proof</u>: Since (2.11) is u.a.s. there exist positive numbers m_1 and m_2 such that its transition matrix $\phi(t,\tau)$ satisfies

$$\|\phi(t,\tau)\| \leq m_1 e^{-m_2(t-\tau)} \qquad \text{for all } t \geq \tau \text{ and all } \tau \geq 0$$

Therefore from (2.11) we can write

$$\|Z(t)\| \leq m_1 e^{-m_2 t}\|Z(0)\| + \int_0^t m_1 e^{-m_2(t-\tau)}\|\bar{H}\|\,\|\eta(\tau)\|d\tau \tag{2.13}$$

Since A_f is asymptotically stable and $\dot{u}(t)$ is bounded by assumption we set from (2.5)

$$\|\eta(t)\| \leq \alpha_1 e^{-\alpha_2 t/\mu}\|\eta(0)\| + \int_0^t \alpha_1 e^{-\alpha_2\frac{(t-\tau)}{\mu}}\|A_f^{-1}B_f\|\gamma d\tau \tag{2.14}$$

where $\gamma = \sup_{t\geq\tau}\|\dot{u}(\tau)\|$ and α_1, α_2 are positive constants. From (2.13) and (2.14) we have

$$\begin{aligned}\|Z(t)\| \leq \mu\gamma \frac{m_1}{m_2}\frac{\alpha_1}{\alpha_2}\|\bar{H}\|\|A_f^{-1}B_f\| + m_1 e^{-m_2 t}\{\|Z(0)\| - \alpha_1 m_1 \frac{\|H\|\|\eta(0)\|}{(m_2-\frac{\alpha_2}{\mu})} \\ - \mu\gamma\frac{\alpha_1}{\alpha_2}\frac{m_1}{m_2}\|H\|\|A_f^{-1}B_f\| + \mu\gamma\frac{\alpha_1}{\alpha_2}m_1\frac{\|A_f^{-1}B_f\|\|H\|}{(m_2-\frac{\alpha_2}{\mu})}\} + \frac{\alpha_1 m_1\|H\|}{(m_2-\frac{\alpha_2}{\mu})}e^{-\alpha_2 t/\mu}\{\|\eta(0)\| \\ - \mu\gamma\frac{\|A_f^{-1}B_f\|}{\alpha_2}\}\end{aligned} \tag{2.15}$$

and (2.12) follows as $t \to \infty$.

Bound (2.12) is convenient because it will be shown that factor $\mu\gamma \frac{m_1}{m_2} \frac{\alpha_1}{\alpha_2} \|A_f^{-1}B_f\| \|H\|$ remains the same for all adaptive schemes considered in this paper. The dependence of (2.12) on μ confirms that the schemes are robust because $Z \to 0$ as $\mu \to 0$. The factor $\frac{\alpha_1}{\alpha_2} \|A_f^{-1}B_f\| \|H\|$ is determined by the parasitics, m_1 depends on the initial error $Z(0)$ and m_2 depends on the rate of convergence without modeling error. The presence of parameter γ characterizing the input will be shown to have a crucial effect on the parameter and state errors. In Sections 3, 4, 5 and 6 we establish u.a.s. of the homogeneous part of (2.11) and derive specific forms of the bound (2.12) for six different adaptive schemes.

3. IDENTIFICATION [1],[2]

It is desired to identify the pair (A,B) in (2.4) by using an nth order model and assuming that the state vector x is available for measurement. The presence of the parasitic input $H\eta$ is disregarded in the design of the identification algorithm.

The nth order model for the identification of the pair (A,B) is given by [1], [2]

$$\dot{x}_m = K(x_m - x) + A_m(t)x + B_m(t)u \tag{3.1}$$

where K is a stable matrix and the adaptive laws for adjusting $A_m(t)$ and $B_m(t)$ are

$$\dot{\phi} = -\Gamma_1 e x' \tag{3.2}$$

$$\dot{\Psi} = -\Gamma_2 e u' \tag{3.3}$$

where $\phi \triangleq A_m(t) - A$, $\Psi \triangleq B_m(t) - B$ and $e \triangleq x_m - x$ are the parameter and state errors and $\Gamma_1 = \Gamma_1' > 0$, $\Gamma_2 = \Gamma_2' > 0$.

In the absence of parasitics ($\eta = 0$) it is shown in [2] that if $u(t)$ is sufficiently rich for an nth order plant (ie the components of $u(t)$ are linearly independent and each component contains at least $\frac{n+1}{2}$ distinct frequencies) then $e, \phi, \Psi \to 0$ as $t \to \infty$. The stability of the identification algorithm in the presence of parasitics is equivalent to the stability of the following system.

$$\dot{e} = Ke + \phi x + \Psi u - H\eta \tag{3.5}$$

$$\dot{\phi} = -\Gamma_1 e x' \tag{3.6}$$

$$\dot{\Psi} = -\Gamma_2 e u' \tag{3.7}$$

where (3.5) is obtained by subtracting (2.4) from (3.1). To express (3.5)-(3.7) as a linear time varying equation in the form of (2.11) we define $Z(t) = [e', \bar{\phi}', \bar{\Psi}']'$ where $\bar{\phi} = [\phi_1, \phi_2, \ldots, \phi_n]'$, $\bar{\Psi} = [\Psi_1, \Psi_2, \ldots, \Psi_n]'$ and ϕ_i, Ψ_i are the ith rows of ϕ and Ψ respectively. Then we denote

$$A_\eta(t) = \left[\begin{array}{c|cc|cc} K & \begin{matrix} x' & & 0 \\ 0 & \ddots & x' \end{matrix} & & \begin{matrix} u' & & 0 \\ 0 & \ddots & u' \end{matrix} & \\ \hline -\Gamma_{1x} & & & & \\ \cline{1-1} -\Gamma_{2u} & & 0 & & \end{array}\right], \quad \bar{H} = \left[\begin{array}{c} -H \\ \hline 0 \end{array}\right] \tag{3.8}$$

where

$$\Gamma_{1x} = \begin{bmatrix} \gamma_1^{(1)} x_1 \\ \gamma_2^{(1)} x_2 \\ \vdots \\ \gamma_n^{(1)} x_n \end{bmatrix}, \quad \Gamma_{2u} = \begin{bmatrix} \gamma_1^{(2)} u_1 \\ \gamma_2^{(2)} u_2 \\ \vdots \\ \gamma_r^{(2)} u_r \end{bmatrix}$$

and $\gamma_j^{(i)}$ is the jth row of Γ_i, $i = 1,2$. To apply lemma 1 we now investigate whether

$$\dot{Z}(t) = A_\eta(t)Z(t) \tag{3.9}$$

is u.a.s. or not. The stability of (3.9) can be easily established by choosing the same Lyapunov function as in [1], [2] for the case without parasitics ($\eta = 0$). However for u.a.s. the components of the vector $\begin{bmatrix} x \\ u \end{bmatrix}$ have to be linearly independent functions of time. A sufficient condition for the case $\eta = 0$ is that the components of u(t) be linearly independent and each component contains at least $\frac{n+1}{2}$ distinct frequencies. In our case x depends on η which in turn depends on $\dot{u}$. Thus in some cases η might destroy the "richness" property of u. This can be avoided by choosing u such that the components of the vector $[x',u',\eta']'$ are linearly independent functions of time, for which a sufficient condition is that the components of $[u',\dot{u}']'$ be linearly independent and each component of u be the sum of sinusoids with at least $\frac{n+m+1}{2}$ distinct frequencies. This implies that the components of $[x',u']$ are linearly independent functions of time and (3.9) is u.a.s. Thus, lemma 1 immediately furnishes bound (3.4), since $\|H\| = \|\bar{H}\|$. We summarize this result in theorem 1.

<u>Theorem 1</u>: If u is sufficiently rich for the (n+m)-th order plant and the components of $[u',\dot{u}']'$ are linearly independent then the identification algorithm (3.1)-(3.3) is stable in the sense that the composite error Z(t) is bounded. The bound is of order of μ and is given by (2.12).

<u>Remark 1</u>: To guarantee the u.a.s. of (3.9) it is sufficient to make u sufficiently rich for the highest suspected order of the actual plant. Although this is a feasible approach in most applications, there is a considerable "overkill" in requiring this richness. In fact it can be shown that the system (3.9) will remain u.a.s. for almost all u which are sufficiently rich for the nth order dominant part of the plant only. For example each component of u can contain any $\frac{n+1}{2}$ distinct frequencies except for a particular combination for which the condition of linear independence of x and u can be lost.

4. MINIMAL FORM ADAPTIVE OBSERVERS [7]

The plant (2.4)-(2.6) is assumed to be single input single output and an nth order adaptive observer is designed to estimate the state vector x of the dominant part of the plant and to identify the triple (A,B,C) or its equivalent. The presence of the parasitic input $H\eta$ is disregarded during the design. The stability of the adaptive observer operating on the real plant in the presence of the parasitic input is then analyzed. Two different types of minimal adaptive observers are considered separately, Case 1 and Case 2.

Case 1. Adaptive observer [6]

Without loss of generality let us assume that the model of the dominant part of the plant (2.4) is in the observable canonical form

$$\dot{x} = \left[\begin{array}{c|c} -\alpha & I \\ \hline & 0 \end{array}\right] x + Bu + H\eta \tag{4.1}$$

$$\mu\dot{\eta} = A_f\eta + \mu A_f^{-1}B_f\dot{u} \tag{4.2}$$

$$y = c'x = [1\ 0\ ..\ 0]x = x_1 \tag{4.3}$$

The algorithm [6] for the nth order adaptive observer based on the dominant part (4.1), (4.3) without the parasitics ($\eta = 0$ in (4.1)) is given by the equations (4.4) through (4.11), below. The observer equation is

$$\dot{z} = Kz + [k - \hat{a}(t)]y + \hat{b}(t)u + w + r \tag{4.4}$$

$$\hat{y} = c'z = z_1 \tag{4.5}$$

where w and r are auxiliary signals formed by the output error $e_1 \triangleq \hat{y} - y$ and the components

$$v_i = \frac{s^{n-i}}{s^{n-1} + d_2 s^{n-2} + \ldots + d_n}\, x_1, \quad q_i = \frac{s^{n-i}}{s^{n-1} + d_2 s^{n-2} + \ldots + d_n}\, u \tag{4.6}$$

of the vectors v and q as follows:

$$w = -e_1 \begin{bmatrix} 0 \\ v'\Gamma A_2 v \\ \vdots \\ v'\Gamma A_j v \\ \vdots \\ v'\Gamma A_n v \end{bmatrix}, \quad r = -e_1 \begin{bmatrix} 0 \\ q'MA_2 q \\ \vdots \\ q'MA_j q \\ \vdots \\ q'MA_n q \end{bmatrix} \tag{4.7}$$

Matrices A_j are

$$A_j = \begin{array}{c} \begin{array}{cccccccccccc} & \multicolumn{6}{c}{\overbrace{\qquad\qquad\qquad\qquad\qquad\qquad}^{n-j+1}} & \multicolumn{5}{c}{\overbrace{\qquad\qquad\qquad\qquad}^{j-2}} \end{array} \\ \left[\begin{array}{cccccccccccc} 0 & -d_j & -d_{j+1} & \cdot & \cdot & \cdot & -d_n & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & -d_j & \cdot & \cdot & \cdot & -d_{n-1} & -d_n & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & -d_j & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -d_n \\ 0 & 1 & d_2 & \cdot & \cdot & \cdot & d_{j-1} & 0 & 0 & \cdot & \cdot & 0 \\ 0 & 0 & 1 & d_2 & \cdot & \cdot & d_{j-2} & -d_{j-1} & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot & 1 & d_2 & d_3 & \cdot & \cdot & \cdot & -d_{j-1} \end{array}\right] \end{array} \begin{array}{l} \\ \left.\vphantom{\begin{array}{c}0\\0\\0\\0\end{array}}\right\} j-1 \\ \left.\vphantom{\begin{array}{c}0\\0\\0\\0\end{array}}\right\} n-j+1 \end{array} \tag{4.8}$$

and $\Gamma = \Gamma' > 0$, $M = M' > 0$ while

$$K = \left[\begin{array}{c|c} k & \begin{array}{c} I \\ \hline 0 \end{array} \end{array}\right], \quad d = \begin{bmatrix} 1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} \tag{4.9}$$

are chosen such that $c'(sI-K)^{-1}d$ is positive real. The adaptive laws for updating the estimated parameters are given by

$$\dot{\phi} = -\Gamma e_1 u = -\dot{\hat{a}}(t) \qquad (4.10)$$

$$\dot{\Psi} = -Me_1 q = \dot{\hat{b}}(t) \qquad (4.11)$$

where $\phi \triangleq \alpha - \hat{a}(t)$ and $\Psi \triangleq \hat{b}(t) - B$ are the parameter errors.

Case 2. Adaptive observer [5], [7].

The following "modal" canonical form is chosen for the dominant part of the plant (2.4)

$$\dot{x} = \left[a \left| \begin{array}{c} h' \\ \hline \Lambda \end{array} \right. \right] x + Bu + H\eta \qquad (4.12)$$

$$\mu\dot{\eta} = A_f \eta + \mu A_f^{-1} B_f \dot{u} \qquad (4.13)$$

$$y = c'x = x_1 \qquad (4.14)$$

where $h' = [1\ 1\ \ldots\ 1]$, Λ is an $(n-1)x(n-1)$ diagonal matrix with arbitrary but known constant and negative diagonal elements $-\lambda_i$ $(i = 2\ldots n)$ and a, B are the unknown vectors to be identified. It is shown in [5] that any completely observable system can be represented in this "modal" canonical form. The structure of the adaptive observer based on (4.12) with $\eta = 0$ is summarized in the equations (4.15) through (4.20). The adaptive observer equation is

$$\dot{z} = Kz + (k - \hat{a}(t))y + \hat{b}(t)u + w + r \qquad (4.15)$$

$$\hat{y} = c'z = z_1 \qquad (4.16)$$

where w and r are auxiliary signals formed by the derivatives of the parameter error components and the components

$$v_i = \frac{1}{s + \lambda_i} x_1, \quad q_i = \frac{1}{s + \lambda_i} u \quad (i=2,..n) \qquad (4.17)$$

of the vectors v and q as follows

$$w = -\begin{bmatrix} 0 \\ \dot{\phi}_2 v \\ \vdots \\ \dot{\phi}_n v \end{bmatrix}, \quad r = \begin{bmatrix} 0 \\ \dot{\Psi}_2 q \\ \vdots \\ \dot{\Psi}_n q \end{bmatrix} \tag{4.18}$$

Moreover $K = \left[\begin{array}{c|c} -\lambda_1 & h' \\ 0 & \\ \cline{2-2} \vdots & \Lambda \\ 0 & \end{array}\right]$ is stable, the transfer function $c'(sI-K)^{-1}d = \frac{1}{s+\lambda_1}$ is strictly positive real, and $d = (1\ 0\ \ldots\ 0)'$. Note that the first components of the vectors v and q are $v_1 = x_1$, $q_1 = u$, respectively. The adaptive laws for adjusting the parameters are

$$\dot{\phi} = -\Gamma e_1 u = \dot{\hat{a}}(t) \tag{4.19}$$

$$\dot{\Psi} = -Me_1 q = \dot{\hat{b}}(t) \tag{4.20}$$

It is shown in Appendix I that the stability of the adaptive observers in case 1 and 2 in the presence of parasitics is equivalent to the stability of the following set of differential equations.

$$\dot{\varepsilon} = K\varepsilon + d[\phi' v + \Psi' q] - H\eta \tag{4.21}$$

$$\epsilon_1 = e_1 \tag{4.22}$$

$$\dot{\phi} = -\Gamma\epsilon_1 v \tag{4.23}$$

$$\dot{\Psi} = -M\varepsilon_1 q \tag{4.24}$$

where K, d, v and q are defined differently in case 1 and 2

It is shown in [7] that if u(t) is sufficiently rich for an nth order plant (ie it contains at least n-distinct frequencies) without parasitics ($H\eta = 0$) then the system (4.21)-(4.24) is u.a.s. To study the stability of the algorithm with parasitics ($H\eta \neq 0$) we express (4.21)-(4.24) in the form of (2.11)

by introducing the composite error vector $Z(t) = [\varepsilon',\phi',\Psi']'$ where .

$$A_\eta(t) = \left[\begin{array}{c|c|c|c} \multicolumn{2}{c|}{K} & du' & dq' \\ \hline -\Gamma u & & & \\ \cline{1-1} -Mq & 0 & \multicolumn{2}{c}{0} \end{array}\right], \quad \bar{H} = \begin{bmatrix} -H \\ \hline 0 \end{bmatrix} \tag{4.26}$$

The stability of the homogeneous part of (2.11) can be shown using the same Lyapunov function as in the case without parasitics [7], [12] and it is not influenced by the fact that $A_\eta(t)$ depends on η. However the proof of u.a.s. is different.

Theorem 2: If u(t) is sufficiently rich for the (n+m)th order plant then the homogeneous part of (2.11) is u.a.s. and the error vector Z(t) is bounded by (2.12).

Proof: For u.a.s. of the homogeneous part of (2.11) a condition has to be imposed on u(t). This is a consequence of the fact that the components of the vector $[v',q']'$ have to be linearly independent functions of time. This implies that the components of $[x',u]'$ have to possess this independence property . It can be shown that if u is sufficiently rich for the (n+m)-th plant, then the components of $[x',u,\dot{\eta}']'$ are linearly independent functions of time. This implies that the components of $[x',u]'$ are linearly independent functions of time hence the homogeneous part of (2.11) is u.a.s. Thus using lemma 1 (2.12) follows.

Remark 2: As in Remark 1 in this it can also be shown that u.a.s. for the homogeneous part of (2.11) can be achieved for almost all u which are sufficiently rich for the nth order plant. That is if u contains at least n distinct frequencies except for a particular combination for which the condition of linear independence of the components of $[x',u]'$ can be lost then the homogeneous part of (2.11) is u.a.s.

5. NON-MINIMAL ADAPTIVE OBSERVER [4], [7], [8]

A non-minimal state representation of the plant (2.4)-(2.6) is (see Appendix II and Fig. 2)

$$\begin{bmatrix} \dot{y} \\ \dot{Z}_s \\ \dot{W}_s \\ \dot{R}_s \end{bmatrix} = \begin{bmatrix} a_1 & a_s' & b_s' & h' \\ h & \Lambda & 0 & 0 \\ 0 & 0 & \Lambda & 0 \\ 0 & 0 & 0 & \Lambda \end{bmatrix} \begin{bmatrix} y \\ Z_s \\ W_s \\ R_s \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ h \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ T_s \end{bmatrix} H\eta + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} h' e^{\Lambda t} \bar{x}(0) \tag{5.1}$$

$$\bar{y} = [1 \quad 0 \quad \dots \quad 0] \begin{bmatrix} y \\ Z_s \\ W_s \\ R_s \end{bmatrix} \tag{5.2}$$

$$\mu\dot{\eta} = A_f\eta + \mu A_f^{-1} B_f \dot{u} \tag{5.3}$$

where $h' = [1 \quad 1 \quad \dots \quad 1]$ and Λ is as defined in Section 4.

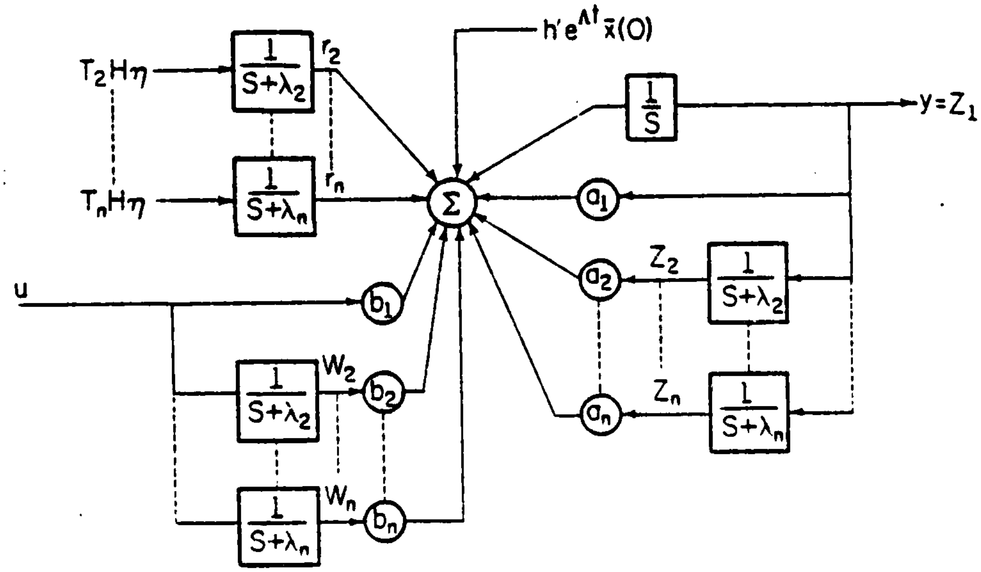

Fig. 2. Non-minimal representation of the dominant part of the plant.

The structure of the adaptive observer for (5.1), (5.2) in the absence of parasitics (ie $\eta = 0$, $R_s = 0$) is given in [4], [7] and [8] and the basic equations are reviewed below. The observer equations are:

$$\dot{\hat{y}} = \hat{a}_1(t)\hat{y} + a_s'\hat{Z}_s + \hat{b}_1(t)u + \hat{b}'(t)W_s - \lambda_1(\hat{y}-y) \tag{5.4}$$

$$\dot{\hat{Z}}_s = \Lambda\hat{Z}_s + hy \tag{5.5}$$

$$\dot{\hat{W}}_s = \Lambda\hat{W}_s + hu \tag{5.6}$$

where $\hat{y}(0) = 0$, $\hat{Z}_s(0) = 0$, $\hat{W}_s(0) = 0$. The adaptive laws for adjusting the unknown parameters are given by:

$$\dot{\phi} = -\Gamma e_1 v \tag{5.7}$$

$$\dot{\Psi} = -Me_1 p \tag{5.8}$$

where $\phi \triangleq [(\hat{a}_1(t)-a_1), (\hat{a}(t)-a_s)']$, $\Psi \triangleq [(\hat{b}_1(t)-b_1), (\hat{b}(t)-b_s)']$ and $v = [y,\hat{Z}_s']'$, $p = [u,\hat{W}_s']'$. The stability of the non-minimal adaptive observer described by (5.4)-(5.8) in the presence of parasitics is equivalent to the stability of

$$\dot{e}_1 = -\lambda_1 e_1 + d[v'\phi + q'\Psi] - h'R_s - h'e^{\Lambda t}\bar{x}(0) \tag{5.9}$$

$$\dot{\phi} = -\Gamma e_1 v \tag{5.10}$$

$$\dot{\Psi} = -Me_1 p \tag{5.11}$$

where (5.9) is obtained by substracting (5.4) from (5.2).

<u>Theorem 3</u>: If $u(t)$ is sufficiently rich for the (n+m)th order plant then the vector $Z(t) = [e_1,\phi',\Psi']'$ is bounded, the bound is of order of μ and is given by

$$\lim_{t\to\infty} \|Z(t)\| \leq \mu\gamma \frac{m_1}{m_2}\frac{\alpha_1}{\alpha_2} \|A_f^{-1}B_f\|\|h\|\|T_sH\| \frac{\xi_1}{\xi_2} \tag{5.12}$$

Proof: We express (5.9) through (5.11) in a compact form similar to (2.11)

$$\dot{Z}(t) = A_\eta(t)Z(t) + A_R R_s(t) \tag{5.13}$$

where

$$A_\eta(t) = \left[\begin{array}{c|cc} -\lambda_1 & dv & dp \\ \hline -\Gamma v & & \\ & 0 & \\ -Mp & & \end{array}\right], \quad A_R = \left[\begin{array}{c} -h' \\ \hline 0 \end{array}\right] \tag{5.14}$$

and note that $A_\eta(t)$ depends on η through v.

The stability of the homogeneous part of (5.13) is proved in [4], [7], [8], [12] by choosing an appropriate Lyapunov function. However for u.a.s. the input u(t) has to belong to a special class of inputs. When $\eta = 0$ it is shown in [7] that a sufficient condition for u.a.s. of the homogeneous part of (5.13) is that u has to be sufficiently "rich" for an nth order plant. Using the same argument as in the proof of Theorem 2 it can be shown that if u is sufficiently rich for the (n+m)th plant the u.a.s. of the homogeneous part of (5.13) is assured. As in Lemma 1 u.a.s. of (5.13) implies that

$$\|Z(t)\| \leq m_1 e^{-m_2 t}\|Z(0)\| + \int_0^t m_1 e^{-m_2(t-\tau)} \|A_R\|\|R_s(\tau)\| d\tau \tag{5.15}$$

From (5.1) for some positive constants ξ_1, ξ_2 we have

$$\|R_s(t)\| \leq \int_0^t \xi_1 e^{-\xi_2(t-\tau)} \|T_sH\|\|\eta(\tau)\| d\tau \tag{5.16}$$

from (2.4), (5.15) and (5.16)

$$
\begin{aligned}
\|Z(t)\| < m_1 e^{-m_2 t}[\|Z(0)\| - \mu \frac{\gamma}{m_2}\frac{\alpha_1}{\alpha_2}\frac{\xi_1}{\xi_2}\|A_f^{-1}B_f\|\|A_R\|\|T_s H\|] \\
+ \frac{(\ell^{-\xi_2 t} - \ell^{-m_2 t})}{(m_2 - \xi_2)} m_1\xi_1\|A_R\|[\|R_s(0)\| - \mu\gamma\frac{\alpha_1}{\alpha_2}\|A_f^{-1}B_f\|\|T_s H\|(\frac{1}{\xi_2} - \frac{1}{\xi_2 - \frac{\alpha_2}{\mu}})] \\
+ \frac{(\ell^{-\alpha_2 t/\mu} - \ell^{-m_2 t})}{(m_2 - \frac{\alpha_2}{\mu})}\alpha_1\xi_1 m_1 \frac{\|A_R\|\|T_s H\|}{(\xi_2 - \frac{\alpha_2}{\mu})}[\|\eta(0)\| - \mu\gamma/\alpha_2\|A_f^{-1}B_f\|] \\
+ \mu\gamma\frac{\alpha_1}{\alpha_2}\frac{m_1}{m_2}\|A_f^{-1}B_f\|\|T_s H\|\|A_R\|\frac{\xi_1}{\xi_2}
\end{aligned}
\tag{5.17}
$$

As $t\to\infty$ $\lim_{t\to\infty}\|Z(t)\| < \mu\gamma\frac{m_1}{m_2}\frac{\alpha_1}{\alpha_2}\frac{\xi_1}{\xi_2}\|A_f^{-1}B_f\|\|T_s H\|\|A_R\|$. Note that $\|A_R\| = \|h\|$ hence (5.12).

It is pointed out that Remark 2 applies to this case as well.

6. PARAMETRIZED OBSERVER [9]

It is shown in Appendix III that the plant equations, (4.1)-(4.3) can be written as

$$\dot{M}(t) = FM(t) + [Iy, Iu] \qquad M(0) = 0 \tag{6.1}$$

$$X(t) = M(t)p^* + EXP(F\,t)X(0) + D(t) \tag{6.2}$$

$$\dot{D}(t) = FD(t) + H\eta(t) \qquad D(0) = 0 \tag{6.3}$$

$$\mu\dot{\eta} = A_f\eta(t) - \mu A_f^{-1}B_f\dot{u} \tag{6.4}$$

$$y = c'x = x_1 \tag{6.5}$$

where p* contains all the unknown parameters of the matrix A and vector B in (4.1). The adaptive observer [9] for observing the state x and estimating p* when D(t) = 0 is reviewed below. The observer equation is

$$\hat{x}(t) = M(t)\hat{p}(t) + EXP(F\,t)\hat{x}(0) \tag{6.6}$$

$$\hat{y}(t) = c'\hat{x} \tag{6.7}$$

The form of the adaptive laws for updating the unknown vector $\hat{p}$ depend on the particular criterion chosen for minimizing the output error $e_1 \triangleq \hat{y} - \hat{y}$. For the first adaptation scheme the error criterion is

$$\zeta_1 = \frac{1}{2} e_1^2 \tag{6.8}$$

and the adaptive law [9], [10] is given by

$$\dot{\hat{p}}(t) = -GM'(t)c\ e_1 \tag{6.9}$$

where $$G = G^T > 0.$$

For the second adaptive scheme the error criterion is

$$\zeta_2 = \int_0^t [c'M(\tau)\hat{p}(t) + c'EXP[F\tau]\hat{x}(0) - y(\tau)]e^{-q(t-\tau)}d\tau \tag{6.10}$$

where q is a positive constant, and the adaptive laws [9] are

$$\dot{\hat{p}} = -G[R(t)\hat{p}(t) + r(t)] \tag{6.11}$$

$$\dot{R}(t) = -qR(t) + M'(t)cc'M(t) \qquad R(0) = 0 \tag{6.12}$$

$$\dot{r}(t) = -qr(t) + M'(t)c[c'EXP[Ft]\hat{x}(0) - y(t)] \qquad r(0) = 0 \tag{6.13}$$

We now analyze the stability of each of the two adaptation schemes in the presence of the parasitic input. We also derive bounds for the composite identification and observation error

$$Z(t) = \begin{bmatrix} \hat{x}(t) - x(t) \\ \hat{p}(t) - p^* \end{bmatrix} = \begin{bmatrix} e(t) \\ \Delta p(t) \end{bmatrix} \tag{6.14}$$

In the first adaptation scheme the state error e and parameter error Δp satisfy

$$\dot{e} = -M(t)GM'(t)cc'e + [FM(t) + (Iy,Iu)]\Delta p + \exp(Ft)Fe(0) - FD(t) - H\eta(t) \tag{6.15}$$

$$\Delta\dot{p} = -GM'(t)cc'e \tag{6.16}$$

Combining (6.17) and (6.18) the equation for the composite error Z(t) is

$$\dot{Z}(t) = A_\eta(t)Z(t) + B_\eta(t) \tag{6.17}$$

where

$$A_\eta(t) = \begin{bmatrix} -M(t)GM'(t)cc' & FM(t) + [Iy,Iu] \\ -GM'(t)cc' & 0 \end{bmatrix} \tag{6.18}$$

and

$$B_\eta(t) = \begin{bmatrix} e^{Ft}Fe(0) - H\eta(t) - FD(t) \\ 0 \end{bmatrix} \tag{6.19}$$

The following Theorem establishes the condition for the u.a.s. of the homogeneous part of (6.17) and gives a bound for Z(t).

<u>Theorem 4</u>: If u(t) is sufficiently rich for an (n+m)th order plant then the nth order adaptive observer given by (6.6), (6.7), (6.9) is stable in the presence of the parasitic part (6.4) of the plant (6.1)-(6.5) in the sense that the composite error vector Z is bounded. A bound on Z as $t\to\infty$ is of order of μ and is given by

$$\lim_{t\to\infty} \|Z\| < \mu\gamma \frac{\alpha_1}{\alpha_2}\frac{m_1}{m_2}\|H\|\|A_f^{-1}B_f\|[1 + \frac{f_1}{f_2}\|F\|] \tag{6.20}$$

<u>Proof</u>: If u is sufficiently rich for the (n+m)th plant then the components of

$M'(t)c$ are linearly independent functions of time [9]. This implies that there exist constants k_1, k_2 and T such that

$$0 < K_1 I \leq \int_t^{t+T} M'(\tau)cc'M(\tau)d\tau \leq K_2 I \quad \text{for all } t \geq 0 \tag{6.21}$$

is satisfied. Using the same proof as in Theorem 1 of [9] it can be shown that if (6.21) is satisfied then the homogeneous part of (6.17) is u.a.s. with the rate of convergence no less than m_2 where

$$m_2 = \frac{K_1 \min \lambda[G]}{[1 + nK_2 \max \lambda[G]]^2} \tag{6.22}$$

Thus from (6.17)

$$\|Z(t)\| \leq m_1 \ell^{-m_2 t} \|Z(0)\| + \int_0^t m_1 \ell^{-m_2(t-\tau)} \|B_\eta(\tau)\| d\tau \tag{6.23}$$

$$\|B_\eta(t)\| \leq f_1 \ell^{-f_2 t} \|Fe(0)\| + \|H\| \|\eta(t)\| + \|F\| \|D(t)\| \tag{6.24}$$

and

$$\|D(t)\| \leq \int_0^t f_1 e^{-f_2(t-\tau)} \|H\| \|\eta(\tau)\| d\tau \tag{6.25}$$

From (6.23), (6.24), (6.25) and (3.4)

$$\begin{aligned} \|Z(t)\| \leq\ & m_1 \ell^{-m_2 t} \|Z(0)\| + p_1 m_1 \frac{(\ell^{-f_2 t} - \ell^{-m_2 t})}{(m_2 - f_2)} \\ & + p_2 m_2 \frac{(\ell^{-\alpha_2 t/\mu} - \ell^{-m_2 t})}{(m_2 - \frac{\alpha_2}{\mu})} + p_3 m_1 \frac{(1 - \ell^{-m_2 t})}{m_2} \end{aligned} \tag{6.26}$$

where m_1, f_1 are positive constants, $f_2 \leq \min |\lambda[F]|$ and p_1, p_2, p_3 are given by

$$p_1 = [f_1 \|Fe(0)\| + \mu\gamma \frac{\alpha_1}{\alpha_2} \|A_f^{-1} B_f\| f_1 (\frac{1}{f_2 - \frac{\alpha_2}{\mu}} - \frac{\|F\| \|H\|}{f_2} - \alpha_1 f_1 \|F\| \|H\| \frac{\|\eta(0)\|}{(f_2 - \frac{\alpha_2}{\mu})}] \tag{6.27}$$

$$p_2 = [\alpha_1 f_1 \frac{\|F\| \|H\| \|\eta(0)\|}{(f_2 - \frac{\alpha_2}{\mu})} + \alpha_1 \|H\| \|\eta(0)\| - \mu\gamma \frac{\alpha_1}{\alpha_2} \|A_f^{-1} B_f\| (\frac{f_1}{f_2 - \frac{\alpha_2}{\mu}} + \|H\|)] \tag{6.28}$$

$$p_3 = \mu\gamma \frac{\alpha_1}{\alpha_2} \|A_f^{-1}B_f\|\|H\|[1 + \frac{f_1}{f_2}\|F\|] \tag{6.29}$$

As $t\to\infty$ (6.20) follows.

In the second adaptation scheme the state and parameter errors satisfy

$$\dot{e} = [FM(t) + (Iy,Iu) - M(t)GR(t)]\Delta p - M(t)G \int_0^t M'(\tau)cc'\ell^{F\tau}e(0)\ell^{-q(t-\tau)}d\tau$$
$$+ M(t)G \int_0^t M'(\tau)cc'D(\tau)\ell^{-q(t-\tau)}d\tau + \ell^{Ft}Fe(0) - FD(t) - H\eta(t) \tag{6.30}$$

and

$$\dot{\Delta p} = -GR(t)\Delta p - G \int_0^t M'(\tau)cc'\ell^{F\tau}e(0)\ell^{-q(t-\tau)}d\tau + G \int_0^t M'(\tau)cc'D(\tau)\ell^{-q(t-\tau)}d\tau \tag{6.31}$$

respectively.

By defining the composite error Z as in (6.14) it can be easily shown that the stability of the second adaptation scheme is equivalent to the stability of

$$\dot{Z} = A_\eta(t)Z + E(t) + C(t) + K(t) \tag{6.32}$$

where

$$A_\eta(t) = \begin{bmatrix} 0 & [FM(t) + (Iy,Iu) - M(t)GR(t)] \\ 0 & -GR(t) \end{bmatrix} \tag{6.33}$$

$$E(t) = \begin{bmatrix} \ell^{Ft}Fe(0) - M(t)G \int_0^t M'(\tau)cc'\ell^{F\tau}e(0)\ell^{-q(t-\tau)} \\ - G \int_0^t M'(\tau)cc'\ell^{F\tau}e(0)\ell^{-q(t-\tau)} \end{bmatrix} \tag{6.34}$$

$$C(t) = \begin{bmatrix} - FD(t) - H\eta(t) + M(t)G \int_0^t M'(\tau)cc'D(\tau)\ell^{-q(t-\tau)}d\tau \\ 0 \end{bmatrix} \tag{6.35}$$

$$K(t) = \begin{bmatrix} 0 \\ G \int_0^t M'(\tau)cc'D(\tau)\ell^{-q(t-\tau)}d\tau \end{bmatrix} \tag{6.36}$$

Note that $E(t)$ vanishes with an exponential rate not less than $\min\{f_2,q\} = w_2$.

The u.a.s. of the homogeneous part of (6.32) and a bound on the composite error $Z(t)$ are established by the following theorem.

Theorem 5: If the input u is sufficiently rich for the $(n+m)$th order plant then the nth order adaptive observer specified by (6.6), (6.7) and (6.11) through (6.13) is stable in the presence of the parasitic part of the plant. The bound on $Z(t)$ as $t\to\infty$ is of order of μ and is given by

$$\lim_{t\to\infty} \|Z(t)\| \leq \mu\gamma \frac{m_1}{m_2}\frac{\alpha_1}{\alpha_2}\|A_f^{-1}B_f\|\|H\|[1+\frac{f_1}{f_2}\|F\| + \frac{f_1^3}{f_2^3}\frac{\|G\|}{q}[(1+\frac{\xi_1}{\xi_2}\|B\|)\delta$$

$$+ \mu\gamma\frac{\alpha_1}{\alpha_2}\frac{\xi_1}{\xi_2}\|A_f^{-1}B_f\|\|H\|]^2 + \frac{f_1^2}{f_2^2}\frac{\|G\|}{q}[(1+\frac{\xi_1}{\xi_2}\|B\|)\delta + \mu\gamma\frac{\alpha_1}{\alpha_2}\frac{\xi_1}{\xi_2}\|A_f^{-1}B_f\|\|H\|]] \qquad (6.37)$$

Proof: Since $u(t)$ is sufficiently rich for the $(n+m)$th order plant the components of $M'(t)c$ are linearly independent [9]. This condition guarantees the existence of constants K and T such that

$$\int_t^{t+T} M'(\tau)cc'M(\tau)d\tau \geq KI > 0 \qquad \text{for all } t \geq 0 \qquad (6.38)$$

In Theorem 3 of [9] it is shown that if (6.38) is satisfied then the homogeneous part of (6.32) is exponentially stable with a rate of convergence $m_2 = \min[q,\theta_2]$ where

$$\theta_2 = K\ell^{-qT}\min\lambda[G] \qquad (6.39)$$

Thus for some positive constants w_1, s_1, ε_1, G_1, ξ_1 and η_1 it can be shown that

$$\|Z(t)\| \leq m_1 e^{-m_2 t}\|Z(0)\| + m_1\int_0^t \ell^{-m_2(t-\tau)}[\|E(\tau)\| + \|C(\tau)\| + \|K(\tau)\|]d\tau \qquad (6.40)$$

$$\|E(t)\| \leq w_1\ell^{-w_2 t} \qquad (6.41)$$

$$\|C(t)\| \le s_1 \ell^{-s_2 t} + \mu\gamma \frac{\alpha_1}{\alpha_2} \|A_f^{-1}B_f\| \|H\| [1 + \frac{f_1}{f_2} \|F\|] + \|M(t)\| \|K(t)\| \tag{6.42}$$

$$\|M(t)\| \le \varepsilon_1 \ell^{-\varepsilon_2 t} + \frac{f_1}{f_2} [(1 + \frac{\xi_1}{\xi_2} \|B\|)\delta + \mu\gamma \frac{\alpha_1}{\alpha_2} \frac{\xi_1}{\xi_2} \|H\| \|A_f^{-1}B_f\|] \tag{6.43}$$

$$\|K(t)\| \le 6_1 e^{-6_2 t} + \mu\gamma \frac{f_1^2}{f_2^2} \frac{\alpha_1}{\alpha_2} \frac{\|G\|}{q} \|A_f^{-1}B_f\| \|H\| [(1 + \frac{\xi_1}{\xi_2} \|B\|)\delta + \mu\gamma \frac{\alpha_1}{\alpha_2} \frac{\xi_1}{\xi_2} \|H\| \|A_f^{-1}B_f\|] \tag{6.44}$$

Then from (6.40) through (6.44)

$$\|Z(t)\| \le n_1 \ell^{-n_2 t} + \mu\gamma \frac{m_1}{m_2} \frac{\alpha_1}{\alpha_2} \|A_f^{-1}B_f\| \|H\| [1 + \frac{f_1}{f_2} \|F\| + \frac{f_1^2}{f_2^2} \frac{\|G\|}{q} ((1 + \frac{\xi_1}{\xi_2} \|B\|)\delta + \mu\gamma \frac{\alpha_1}{\alpha_2} \frac{\xi_1}{\xi_2} \|A_f^{-1}B_f\| \|H\|) + \frac{f_1^3}{f_2^3} \frac{\|G\|}{q} ((1 + \frac{\xi_1}{\xi_2} \|B\|)\delta + \mu\gamma \frac{\alpha_1}{\alpha_2} \frac{\xi_1}{\xi_2} \|A_f^{-1}B_f\| \|H\|)^2] \tag{6.45}$$

where $s_2 = \min[\frac{\alpha_2}{\mu}, f_2]$, $\varepsilon_2 = \min[\frac{\alpha_2}{\mu}, \xi_2, f_2]$

$$6_2 = \min[\varepsilon_2, q], \qquad n_2 = \min[s_2, \varepsilon_2, 6_2],$$

$\xi_2 \le \min|\lambda[A]|$ and $\delta = \sup\ u(t)$ for all $t \ge 0$. As $t \to \infty$ (6.45) reduces to (6.37).

Remark 3: In both first and second adaptation schemes u was required to be sufficiently rich for an (n+m)th order plant in order for conditions (6.21) and (6.38) to be satisfied. However, as it was pointed out in Remark 2, for almost all u sufficiently rich for an nth order plant the results obtained will still be valid.

7. DISCUSSION

In all the error bounds derived in the preceeding sections the common factor is

$$\mu\gamma \frac{m_1}{m_2} \frac{\alpha_1}{\alpha_2} \|A_f^{-1}B_f\| \|H\| \tag{7.1}$$

The most important terms appearing in (7.1) are μ, the "speed ratio" of slow vs fast phenomena, and γ which is a characteristic of the input. The dependence of the error bound on μ shows that the adaptive schemes considered are robust with respect to the parasitics in the sense that as $\mu \to 0$ the error bound goes to zero. We demonstrate the effect of μ and γ by digital simulation of the adaptive observer (Section 4, Case 1) for the plant.

$$\dot{x} = \begin{bmatrix} -5 & 1 \\ -10 & 0 \end{bmatrix} x + \begin{bmatrix} 0.9 \\ 0.5 \end{bmatrix} x_f + \begin{bmatrix} 1.45 \\ 2.25 \end{bmatrix} u \tag{7.2}$$

$$\mu\dot{x}_f = -4x_f - 2u \tag{7.3}$$

$$y = [1 \quad 0]x \tag{7.4}$$

Using the transformation $\eta = x_f + 0.5u$ the plant state equations become

$$\dot{x} = \begin{bmatrix} -5 & 1 \\ -10 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u + \begin{bmatrix} 0.9 \\ 0.5 \end{bmatrix} \eta \tag{7.5}$$

$$\mu\dot{\eta} = -4\eta + 0.5\mu\dot{u} \tag{7.6}$$

$$y = [1 \quad 0]x \tag{7.7}$$

The adaptive observer for (7.5), (7.7) with $\eta = 0$ is

$$\dot{z} = \begin{bmatrix} -6 & 1 \\ -8 & 0 \end{bmatrix} z + \begin{bmatrix} 6 - \hat{a}_1(t) \\ 8 - \hat{a}_2(t) \end{bmatrix} y + \begin{bmatrix} \hat{b}_1(t) \\ \hat{b}_2(t) \end{bmatrix} u - e_1 \begin{bmatrix} 0 \\ v'\Gamma A_2 v \end{bmatrix} - e_1 \begin{bmatrix} 0 \\ q'MA_2 q \end{bmatrix} \tag{7.8}$$

where

$$A_2 = \begin{bmatrix} 0 & -3 \\ 0 & 1 \end{bmatrix}, \ \Gamma = \begin{bmatrix} 140 & 0 \\ 0 & 75 \end{bmatrix}, \ M = \begin{bmatrix} 5 & 0 \\ 0 & 7.8 \end{bmatrix} \text{ and}$$

the components of the signals v and q are generated by

$$\dot{v}_2 = -3v_2 + x_1 = v_1, \quad \dot{q}_2 = -3q_2 + u = q_1 \tag{7.9}$$

The adaptive laws for adjusting the parameters are taken as

$$\dot{\hat{a}}(t) = \Gamma e_1 v, \quad \dot{\hat{b}}(t) = -Me_1 q \tag{7.10}$$

The dependence of the error bound on μ is illustrated in Figs. 3 and 4. For $\mu = 0.2$ and input $u = 5\sin t + 5\sin 2.5t$ that is $\gamma = 18.2$, the observation error e_2 is relatively small. However the parameter errors are significant: 10.4% for $\hat{a}_1(t)$ and 12% for $\hat{b}_1(t)$. Reduction of μ by a factor 4 that is $\mu = 0.05$ results in a reduction of the parameter errors by approximately the same factor as shown in Fig. 4b, c. The observation error e_2 is almost zero in this case (Fig. 4a).

To examine the effect of $\gamma = \sup|\dot{u}(t)|$ on the error bound the value of μ is kept the same as in Fig. 4 but the input is changed to $u = 5\sin t + 15\sin 2.5t$ that is γ is increased to $\gamma = 42.5$. The results obtained are shown in Fig. 5. It is clear that the value of γ is crucial for the error bound. Increasing γ by a factor of 2.3 results in an increase of the parameter error by a factor of about 10. Moreover the observation error although bounded, is oscillatory and not close to zero.

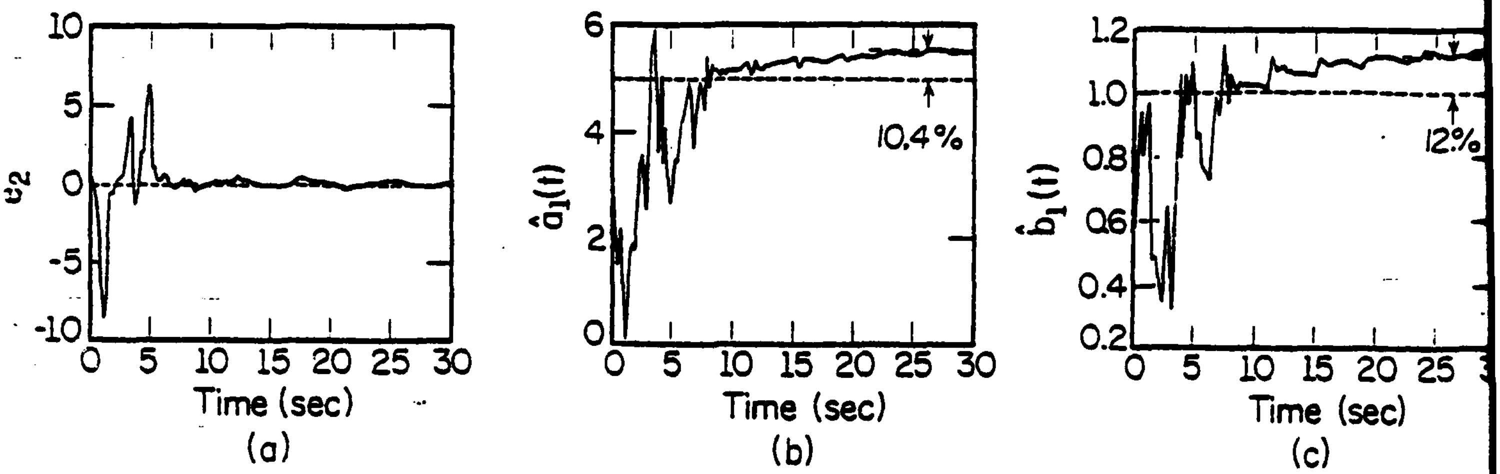

Fig. 3. $\mu = 0.2$, $u = 5 \sin t + 5 \sin 2.5t$ ($\gamma = 18.2$)

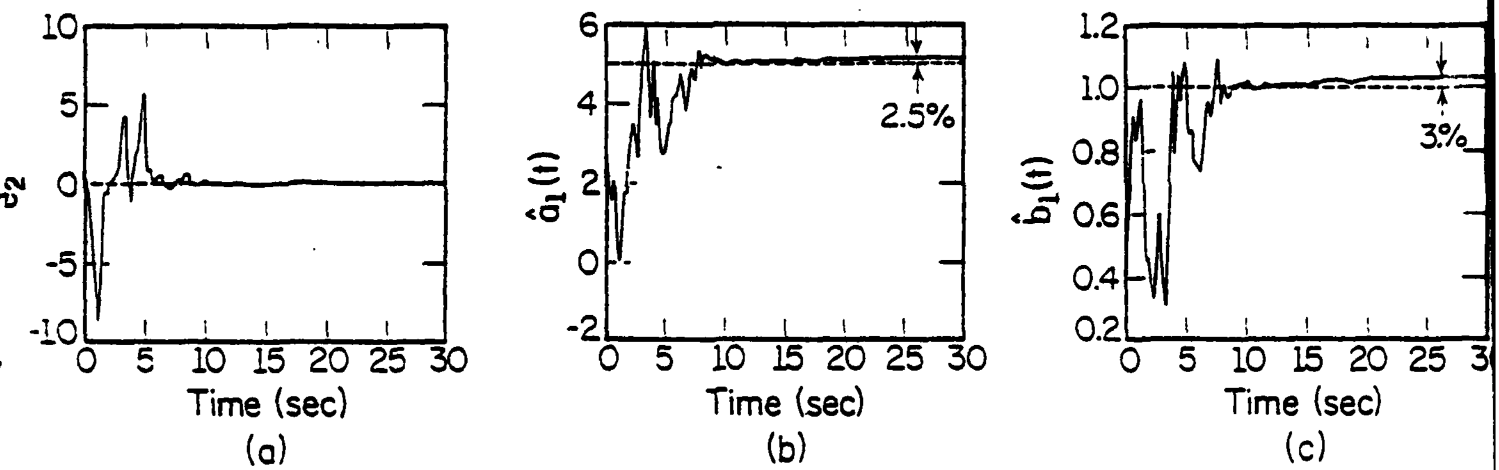

Fig. 4. $\mu = 0.05$, $u = 5 \sin t + 5 \sin 2.5t$ ($\gamma = 18.2$)

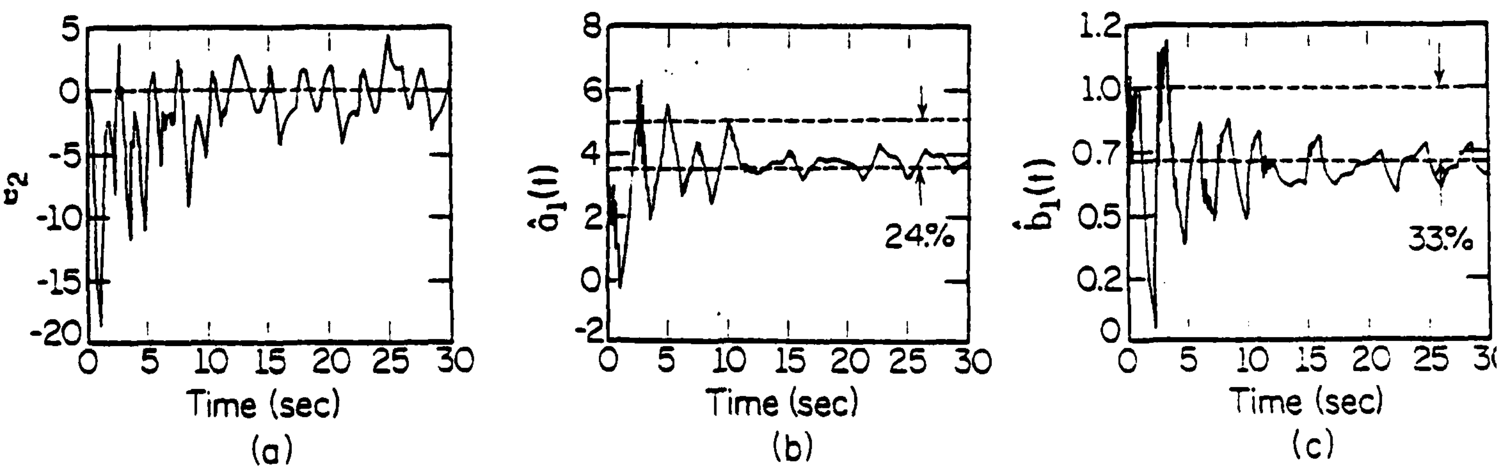

Fig. 5. $\mu = 0.05$, $u = 5 \sin t + 15 \sin 2.5t$ ($\gamma = 42.5$)

The above simulation results show that the choice of the excitation input signal is critical in adaptive schemes with modeling errors. The effect of the input on the error can be even more crucial than the effect of μ. Rapidly varying inputs (high γ) will result into bad estimates of parameters and states. Inputs of this class excite the parasitic part of the plant considerably and have adverse effects on the results of adaptive schemes with modeling error. Caution is needed in selecting an input excitation signal. Our results show that the richness condition, should be satisfied but with an input which has a low value of γ. Slowly varying inputs are appropriate as long as they do not reduce the convergence rate considerably.

Convergence rate m_2 with which $Z(t)$ exponentially decays in $\dot{Z} = A_\eta(t)Z$ is sensitive with respect to γ only for very low values of γ. Thus we can improve the error bound by keeping γ as low as possible for m_2 not to be affected appreciably. From (7.1) it is clear that larger m_2 reduces the error bound. However m_2 depends on the input u and on the adaptive gains and, hence, for low values of γ there is a trade-off between γ and m_2, that is m_2 cannot be improved through the choice of the input. Its improvement by the choice of the adaptive gains can only be done by trial and error since in all the adaptive schemes considered, except in the parametrized observer, m_2 is not explicitely related to the adaptive gains. In the case of the parametrized adaptive observer the expression for m_2 gives more information about the dependence of the error bound on other quantities. For the first adaptation scheme $m_2 = \dfrac{k_1 \min\lambda[G]}{[1 + n\, k_2 \max\lambda[G]]^2}$. In this case the best we can do to improve m_2 and consequently the error bound is to make $\max\lambda[G] = \min\lambda[G]$. The dependence of m_2 on the order n of the dominant part of the plant indicates

that the error bound will be higher for a higher order dominant part of the plant. For the second adaptive scheme $m_2 = \min[q, \theta_2]$ where $\theta_2 = Ke^{-qT} \min \lambda[G]$. In this case m_2 can be increased arbitrarily by increasing G and choosing q appropriately. However the bound for this scheme is proportional to $\|G\|$ and it is not clear whether an increase of m_2 through G will reduce the error. The term m_1 in (7.1) indicates the dependence of the bound on the initial error vector Z(0). The factor $\frac{\alpha_1}{\alpha_2} \|A_f^{-1} B_f\|$ in (7.1) depends on the characteristics of the parasitic part and the term $\|H\|$ is a measure of the coupling between the dominant and parasitic part of the plant. Apart from the common factor (7.1) the bounds obtained for the parametrized adaptive observer are also functions of the characteristics of the observer gain F. The bound for the second scheme is more complicated due to the complexity of that scheme.

8. CONCLUSIONS

The results of the paper show that if the plant is stable, the adaptive schemes considered remain bounded despite the reduction of the model order. The bound on the observation and parameter error is of the order of the singular perturbation parameter μ, and is also a function of the characteristics of the input, the initial parameter and state error and the convergence rate m_2, which would be achieved if there were no modeling error. The dependence of the error on the input characteristics is found to be crucial and the most desirable excitation signals are those which are sufficiently rich, but have a low value of $\gamma = \sup_t |\dot{u}(t)|$. A trade off between γ and m_2 should be made when selecting the input signal. The input does not have to be sufficiently rich for the full order plant. The results of the paper are valid for almost every u(t) sufficiently rich for the nth order dominant part of the plant. Thus u(t) can contain as

many frequencies as required to be rich for the n-order plant except for a particular combination of frequencies for which the richness of $u(t)$ is reduced by the parasitic input $\bar{\eta}(t)$. Extensions of these results to model reference adaptive control and other closed loop adaptive schemes is a topic for future research.

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APPENDIX I

The stability of the adaptive observer in the presence of parasitics for Case 1 and 2 is equivalent to the stability of the following set of differential equations

$$\dot{e} = Ke + \phi x_1 + \Psi u - H\eta + w + r, \quad e_1 = c'e \tag{I.1}$$

$$\dot{\phi} = \Gamma e_1 u \tag{I.2}$$

$$\dot{\Psi} = -Me_1 q \tag{I.3}$$

Proposition: For some vector signals v, q, w and r with $v = G(p)x_1$, $q = G(p)u$, $w = w(\dot{\phi},u)$ and $r = r(\dot{\Psi},q)$ the system (I.1) is input $([x_1,u,\eta']')$-output (e_1) equivalent with the system I.4 provided (c',K) is completely observable

$$\dot{\varepsilon} = K\varepsilon + d[\phi'v + \Psi'q] - H\eta, \quad \varepsilon_1 = c'\varepsilon = e_1 \tag{I.4}$$

Proof: From (I.1), (I.4)

$$c'(pI-K)^{-1}[\phi x_1 + \Psi u + w + r - d(\phi'v + \Psi'q)] \tag{I.5}$$

where 'p' is the d/dt operator. From (I.5) we have

$$\sum_{i=1}^{n} p^{n-i}[\phi_i x_1 + \Psi_i u + w_i + r_i - d_i\phi'v - d_i\Psi'q] = 0 \tag{I.6}$$

where i denotes the ith element of the corresponding vector. (I.6) is satisfied by choosing v, q, w and r as given in Case 1 and Case 2. By considering the two equivalent systems (I.1), (I.2), (I.3) and (I.4), (I.2), (I.3), it is obvious that boundeness of $[\varepsilon',\phi',\Psi']'$ will imply boundeness of $[e',\phi',\Psi']$.

APPENDIX II

From (5.1) and (5.3)

$$\frac{y(s)}{u(s)} = G(s) = G_p(s) + c'(sI-A)^{-1}H\,\frac{\eta(s)}{u(s)}$$

where

$$G_p(s) = c'(sI-A)^{-1}B$$

is the transfer function of the plant when $H\eta = 0$. Let

$$G_p(s) = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \ldots + b_n}{s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \ldots + \alpha_n}$$

where

$$\alpha \triangleq [\alpha_1, \alpha_2, \ldots, \alpha_n]' \text{ and } B \triangleq [b_1, b_2, \ldots, b_n]'$$

are the unknown parameter vectors.

Consider a polynomial $\prod_{i=2}^{n}(s+\lambda_i)$ which is relatively prime to the numerator as well as the denominator polynomials of $G_p(s)$ and $\lambda_i \neq \lambda_j$ for i, $j = 2,3,\ldots,n)$. Dividing the numerator and denominator of G(s) by $\prod_{i=2}^{n}(s+\lambda_i)$ and expanding them into partial fractions we have

$$G(s) = \frac{b_1 + \frac{b_2}{s+\lambda_2} + \ldots + \frac{b_n}{s+\lambda_n}}{s - a_1 - \frac{a_2}{s+\lambda_2} - \ldots - \frac{a_n}{s+\lambda_n}} + \frac{\frac{c'\mathrm{adj}(sI-A)^{-1}H}{\prod_{i=2}^{n}(s+\lambda_i)}\;\frac{\eta(s)}{u(s)}}{s - a_1 - \frac{a_2}{s+\lambda_2} \cdots \frac{a_n}{s+\lambda_n}} \tag{II.1}$$

Note that

$$\frac{c'\mathrm{adj}(sI-A)^{-1}}{\prod_{i=2}^{n}(s+\lambda_i)} = \frac{[s^{n-1}, s^{n-2}, \ldots, 1]}{\prod_{i=2}^{n}(s+\lambda_i)} = \left[\sum_{i=2}^{n}\frac{t_{1i}}{s+\lambda_i}, \sum_{i=2}^{n}\frac{t_{2i}}{s+\lambda_i}, \ldots, \sum_{i=2}^{n}\frac{t_{ni}}{s+\lambda_i}\right] \tag{II.2}$$

II.1 can be written as

$$y(s) = \frac{1}{s}[b_1u(s) + a_1y(s) + \sum_{i=2}^{n} \frac{[b_iu(s) + a_iy(s) + T_iH\eta(s)]}{s + \lambda_i} \qquad \text{(II.3)}$$

where

$$T_i = [t_{1i},\ t_{2i},\ldots,t_{ni}]$$

II.3 is represented in a block diagram form in Figure 2. The block diagram of Figure 2 contains (3n-2) integrators and is a nonminimal realization of the dominant part of the plant.

The term $h'\exp(\Lambda t)\bar{x}(0)$ in Figure 2 is added so that Figure 2 is equivalent to the corresponding figure of a minimum realization of II.3 including initial conditions. Here $\bar{x} = [x_2,x_3,\ldots,x_n]$ and x is the state of the minimal realization based on II.3.

The nonminimal state-space representation given by (6.1), (6.2) can be easily obtained from Figure 2 by defining

$$R_s = [r_2,r_3,\ldots,r_n]', \quad w_s = [w_2,w_3,\ldots,w_n]'$$

$$Z_s = [z_2,z_3,\ldots,z_n]' \text{ and } T_s = [T_2',T_3',\ldots,T_n']'$$

APPENDIX III

Equation (2.4) can be represented as

$$\dot{x} = Fx + gy + Bu + H\eta \tag{III.1}$$

where

$$F = \begin{bmatrix} -f_1 & 1 & 0 \dots & 0 \\ -f_2 & 0 & 1 & \\ \vdots & \vdots & & 1 \\ -f_n & 0 & 0 \dots & 0 \end{bmatrix}, \quad g = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} \tag{III.2}$$

and g satisfies $gc' = A-F$.

From III.1

$$x = \ell^{Ft}x_o + \int_o^t \ell^{F(t-\tau)}[gy(\tau) + Bu(\tau)]d\tau + \int_o^t \ell^{F(t-\tau)}H\eta(\tau)d\tau \tag{III.3}$$

The first convolution integral can be reduced to

$$\int_o^t \ell^{F(t-\tau)}[Iy(\tau),\ Iu(\tau)]d\tau \cdot p^* = M(t)p^* \tag{III.4}$$

where $p^{*\prime} = [g',B']$. Thus (6.1)-(6.3) follows by taking

$$D(t) = \int_o^t \ell^{F(t-\tau)}H\eta(\tau)d\tau \tag{III.5}$$

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